



PROBABILITY-BASED INFERENCE IN A DOMAIN OF PROPORTIONAL REASONING TASKS

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Abstract

Educators and psychologists are increasingly interested in modelling the processes and knowledge structures by which people learn and solve problems. Progress has been made in developing cognitive models in several domains, and in devising observational settings that provide clues about subjects' cognition from this perspective. Less attention has been paid to procedures for inference or decision-making with such information, given that it provides only imperfect information about cognition—in short, test theory for cognitive assessment. This paper describes probability-based inference in this context, and illustrates its application with an example concerning proportional reasoning.

Key words: Bayesian inference, cognitive assessment, inference networks, multiple strategies, proportional reasoning, test theory



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Introduction

The view of human learning rapidly emerging from cognitive and educational psychology emphasizes the active, constructive role of the learner in acquiring knowledge. Learners become more competent not simply by learning more facts and skills, but by configuring and reconfiguring their knowledge; by automating procedures and chunking information to reduce memory loads; and by developing models and strategies that help them discern when and how facts and skills are relevant. Educators have begun to view school learning from this perspective, as a foundation for instruction in both the classroom and intelligent computer-assisted instruction, or intelligent tutoring systems (ITSs). Making educational decisions cast in this framework requires information about students in the same terms. Glaser, Lesgold, and Lajoie state,

Achievement testing as we have defined it is a method of indexing stages of competence through indicators of the level of development of knowledge, skill, and cognitive process. These indicators display stages of performance that have been attained and on which further learning can proceed. They also show forms of error and misconceptions in knowledge that result in inefficient and incomplete knowledge and skill, and that need instructional attention. (Glaser, Lesgold, & Lajoie, 1987, 81)

Standard test theory is designed to characterize students in terms of their tendencies to make correct answers, not in terms of their skills, strategies, and knowledge structures. Yet generalizations of the questions that led to standard test theory arise immediately in the context Glaser and his colleagues describe: How can we design efficient observational settings to gather the data we need? How can we make and justify decisions? How do we evaluate and improve the quality of our efforts? Without a conceptual framework for inference, rigorous answers to these questions are not forthcoming.

This presentation addresses issues in model building and statistical inference in the context of student modelling. The statistical framework is that of inference networks (e.g.,

Pearl, 1988; Andreassen, Jensen, & Olesen, 1990). Ideas are demonstrated with data from a test of proportional reasoning, based on work by Noelting (1980a, 1980b). The observed data are subjects' comparisons of mixtures of juice and water, and their explanations of the strategies by which they arrived at their answers. The cognitive framework builds on Béland's (1989) structural analysis of the task component relationships involved in their solution strategies.

Probability-based Inference in Cognitive Assessment

Comparing the ways experts and novices solve problems in domains such as physics and chess (e.g., Chi, Feltovich & Glaser, 1981) reveals the central importance of knowledge structures—interconnected networks of concepts referred to as "frames" (Minsky, 1975) or "schemas" (Rumelhart, 1980)—that impart meaning to observations and actions. The process of learning is, to a large degree, expanding these structures and, importantly, reconfiguring them to incorporate new and qualitatively different connections as the level of understanding deepens. Researchers in science and mathematics education have focused on identifying key concepts and schemas in these content areas, studying how they are typically acquired (e.g., in mechanics, Clement, 1982; in proportional reasoning, Karplus, Pulos, & Stage, 1983), and constructing observational settings in which students' understandings can be inferred (e.g., van den Heuvel, 1990; McDermott, 1984). A key feature of most of these studies is explaining patterns observed in learners' problem-solving behavior in terms of their knowledge structures. Riley, Greeno, and Heller (1983), for example, explain typical patterns of errors and correct answers in children's word problems in terms of a hierarchy of successively sophisticated procedural models.

Once the relevance of states of understanding to instructional decisions is accepted, one immediately confronts the fact that these states cannot be ascertained with certainty;

they can be inferred only imperfectly from observations of the students' behavior.

Research in subject areas is beginning to provide observational situations (at their simplest form, test items) that tap particular aspects of knowledge structures (e.g., Lesh, Landau, & Hamilton, 1983; Marshall, 1989). Conformable statistical models must be capable of expressing the nature and the strength of evidence that observations convey about knowledge structures. Two kinds of variables are thus involved: those expressing characteristics of an inherently unobservable student model, and those concerning qualities of observable student behavior, the latter of which presumably carry information about the former.

For the special case in which a student is adequately characterized by a single unobservable proficiency variable, a suitable statistical methodology has been developed within the paradigm of standard test theory, most notably under the rubric of item response theory (IRT; see Hambleton, 1989). IRT posits a model for the probability of a correct response to a given test item, as a function of parameters for the examinee's proficiency (often denoted θ) and measurement properties of the item. The IRT model provides the *structure* through which observable responses to test items are related to one another and to the unobservable proficiency variables. Item parameters specify the *degree* or *strength* of relationships within that structure, by quantifying the conditional probabilities of item responses given θ . Observed item responses induce a likelihood function for θ , opening the door to statistical inference and decision-making models. The coupling of probability-based inference with a simple student model for overall proficiency provides the foundation for item development, test construction, adaptive testing, test equating, and validity research—all providing, of course, that "overall proficiency" is sufficient for the job at hand.

Models connecting observations with a broader array of cognitively-motivated unobservable variables have begun to appear in the psychometric literature. Table 1 offers

a sampling. The approach we have begun to follow continues in the same spirit. In any given implementation, the character of unobservable variables and the structure of their interrelationships is derived from the structure and the psychology of the substantive area, with the goal of capturing key distinctions among students. Probability distributions characterize the likelihoods of potential observable variables, given values of the variables in the unobservable student model. The relationship of the observable variables to the unobservable variables characterizes the nature and amount of information they carry.

[Insert Table 1 about here]

Of particular importance is the concept of conditional independence: a set of variables may be interrelated in a population, but independent given the values of another set of variables. In cognitive models, relationships among observed variables are "explained" by inherently unobservable, or latent, variables. Pearl (1988) argues that creating such intervening variables is not merely a technical convenience, but a natural element in human reasoning:

"...conditional independence is not a grace of nature for which we must wait passively, but rather a psychological necessity which we satisfy actively by organizing our knowledge in a specific way. An important tool in such organization is the identification of intermediate variables that induce conditional independence among observables; if such variables are not in our vocabulary, we create them. In medical diagnosis, for instance, when some symptoms directly influence one another, the medical profession invents a name for that interaction (e.g., 'syndrome,' 'complication,' 'pathological state') and treats it as a new auxiliary variable that induces conditional independence; dependency between any two interacting systems is fully attributed to the dependencies of each on the auxiliary variable." (Pearl, 1988, p. 44)

Inference Networks

A heritage of statistical inference under the paradigm described above extends back beyond IRT, to Charles Spearman's (e.g., 1907) early work with latent variables, Sewell Wright's (1934) path analysis, and Paul Lazarsfeld's (1950) latent class models. The resemblance of the inference networks presented below to LISREL diagrams (Jöreskog & Sörbom, 1989) is no accident! The inferential logic of test theory is built around conditional probability relationships—specifically, probabilities of observable variables given theoretically-motivated unobservable variables.

The starting point is a recursive representation of the joint distribution of a set of random variables; that is,

$$p(X_1,...,X_n) = p(X_n|X_{n-1},...,X_1) \ p(X_{n-1}|X_{n-2},...,X_1) \cdots p(X_2|X_1) \ p(X_1)$$

$$= \prod_{j=1}^n p(X_j|X_{j-1},...,X_1) , \qquad (1)$$

where the term for j=1 is defined as simply $p(X_1)$. A recursive representation can be written for any ordering of the variables, but one that exploits conditional independence relationships can be more useful. For example, under an IRT model with one latent proficiency variable θ and three test items, X_1 , X_2 , and X_3 , it is equally valid to write

$$p(X_1, X_2, X_3, \theta) = p(\theta | X_3, X_2, X_1) \ p(X_3 | X_2, X_1) \ p(X_2 | X_1) \ p(X_1)$$
(2)

or

$$p(X_1, X_2, X_3, \theta) = p(X_3 | X_2, X_1, \theta) \ p(X_2 | X_1, \theta) \ p(X_1 | \theta) \ p(\theta).$$
(3)

But (3) simplifies to

$$p(X_1, X_2, X_3, \theta) = p(X_3 | \theta) p(X_2 | \theta) p(X_1 | \theta) p(\theta),$$
(4)

the form that harnesses the power of IRT by expressing test performance as the concatenation of conditionally independent item performances. More generally, (1) can be re-written as

$$p(X_1,...,X_n) = \prod_{j=1}^{n} p(X_j| \text{"parents of } X_j\text{"}),$$
 (5)

where {parents of X_i } is the subset of variables upon which X_i is directly dependent.

Corresponding to the algebraic representation of $p(X_1,...,X_n)$ in (5) is a graphical representation—a directed acyclic graph (DAG). Each variable is a node in the graph; directed arrows run from parents to children, indicating conditional dependence relationships among the variables. In this paper we refer to such a structure or its graphical representation as an inference network. Figure 1 shows the DAGs that correspond to (2) and (4) in the IRT example. Note that the simplified structure is apparent only in the graph for (4). A DAG does not generally reveal conditional independence relationships that might arise under alternative orderings of the variables.

[Insert Figure 1 about here]

Different fields of application emphasize different aspects of inference network representations of systems of variables. In factor analyses of mental tests, for example, one important objective is to find a "simple structure" representation of the relationships among test scores, wherein each test has only a few latent variables as parents (e.g., Thurstone, 1947). In sociological and economic applications, path analysis is used to sort out the direct and indirect effects of selected variables upon others (e.g., Blalock, 1971). In animal husbandry, where genotypes are latent nodes and inherited characteristics of animals are observable, interest lies in the predicted distribution of characteristics of the offspring of potential matings (e.g., Hilden, 1970). In medical diagnosis, disease states and syndromes are unobserved nodes, while symptoms and test results are potential

observables; ascertaining the latter guides diagnosis and treatment decisions (e.g., Andreassen, Jensen, & Olesen, 1990).

The latter arenas have sparked interest in calculating distributions of remaining variables conditional on observed values of a subset. If the topology of the DAG is favorable, such calculations can be carried out in real time in large systems by means of local operations on small subsets of interrelated variables ("cliques") and their intersections. The interested reader is referred to Lauritzen and Spiegelhalter (1988), Pearl (1988), and Shafer and Shenoy (1988) for updating strategies, a kind of generalization of Bayes theorem. The calculations for the following example were carried out with Andersen, Jensen, Olesen, and Jensen's (1989) HUGIN computer program.

The point of this presentation is that inference networks can be constructed around cognitive student models. The analogy to medical applications is sketched in Table 2. A key aspect of the correspondence is the flow of diagnostic reasoning: Theory is expressed in terms of conditional probabilities of observations given theoretically suggested unobservable variables, and it is from this direction that the inference network is constructed. Reasoning in practical applications flows in the opposite direction, as evidence from observations is absorbed, to update belief about the unobservable variables. This necessity of bidirectional reasoning stimulates interest in probability-based inference, as accomplished by the generalizations of Bayes Theorem mentioned above.

[Insert Table 2 about here]

An Inference Network for a Set of Juice-Mixing Tasks

Proportional reasoning is a topic of great current interest among mathematics and science educators, because it constitutes perhaps half of the middle school mathematics curriculum, and is a prerequisite for quantitative aspects of the sciences as well as advanced topics in mathematics.

There is consequently considerable research on this topic among the communities of both

developmental psychology (e.g., Inhelder & Piaget, 1958; Siegler, 1978) and the psychology of mathematics education (e.g., Romberg, Lamon, & Zarinnia, 1988). The network presented here is based on a program of research on the development of proportional reasoning represented by Noelting (1980a; 1980b) and Béland (1990). According to this conceptual framework, subjects' cognitive strategies are explained in terms of the relationships they address vis a vis the structural properties of the items. Development is viewed as a progression through qualitatively distinct levels of understanding.

In order to study the concept of proportion, a basic test of twenty items was devised. Each consisted of predicting the relative taste of two drinks, labeled A and B, which comprised varying numbers of glasses of juice and glasses of water. Each mixture defined an ordered pair, that is (a, b) for the drink labeled A, and (c, d) for the drink labeled B. The first term in each pair defined the number of glasses of juice and the second term defined the number of glasses of water, as shown by the example in Figure 2. In the test, the child had to decide if either A or B would taste juicier, or if both drinks would taste the same. The subjects also had to explain the reasons for their choices by writing a detailed explanation of how they had solved each problem. A total number of 448 subjects, ranging from fourth graders to university freshman, were assessed. Instructions were given and data collected in class groups. The order of item presentation was randomized for each child. To assure that the task was understood, sample items were solved by the classes.

[Insert Figure 2 about here]

An item's components were differentiated as being the varying quantities of juice glasses, which defined the attribute, and water glasses, which defined the complement, in each pair. When a subject attempted to solve an item by constructing transformations between similar terms in both pairs, that is, either between the attribute or the complement in both mixtures, then the relationships were described as scalar. On the other hand, when the transformations were constructed between

the complementary terms within each pair, that is, between the attribute and the complement in a mixture, then the relationships were described as functional. Three qualitatively distinct ordered levels (listed below) were defined as a set of additive and multiplicative relations among the values of these terms. These levels characterize both items and solution strategies: solution strategies, in terms of the kinds of transformations and comparisons they involve; items, by virtue of their structure, in terms of the minimal level required for a correct understanding of the problem. The fact that some strategies led to success with items at one level, but to failure with items at higher levels, indicates a structural discontinuity between these levels. This implies that the transition between these levels involves restructuring, or reconceptualizing, the relationships among task components, in response to the structural properties of the items. The three levels of understanding are as follows.

- Level 1, the *preoperational* level, is characterized by the differentiation and coordination of scalar and functional relationships. For example, one justification for solving the item (2,1) vs. (3,4) was: "Mixture A tastes juicier because the number of juice glasses is greater than the number of water glasses. By comparison, mixture B tastes less juicy because the quantity of water glasses is greater than juice glasses."
- Level 2, the concrete operational level, is characterized by the construction of an equivalence class. For example, to solve the item (2,6) vs. (3,9), the typical justification for the functional operator was: "Both drinks taste alike because there is one glass of juice for three glasses of water, which defines the ratio 1:3 in both pairs."
- Level 3, the *formal operational* level, is characte—ized by the construction of a combinatorial system, building upon the concepts from the previous levels. An item is solved either by the *between* state ratios (common denominator) or the within state ratios (percentage). For example, when a ratio strategy was used to

solve (3,5) vs. (2,3), the typical justification was: "In Mixture A there are three glasses of juice for five glasses of water, a ratio of 9:15. In Mixture B the ratio is 10:15 juice to water. Therefore, B tastes juicier."

The gradual extension of these structures, through exercise and practice, leads to the consolidation of the cognitive strategies as they are applied to solve the increasing complexity of the items within a level. This progression was defined as *stage within level*. Three successive stages, denoted as a, b, and c, were defined within each level. Table 3 summarizes the stages within levels. The reader is referred to Béland (1990) for additional detail and discussion.

[Insert Table 3 about here]

An Overview of the Network

An inference network was constructed on the basis of the data described above, addressing subjects' optimal cognitive stage x level, or the highest stage and level at which they were observed to perform during the course of observation, and the details of their responses to three items, one at each level. This section introduces the network. The following section describes the variables in more detail, and discusses the specification of conditional probabilities. The section after that gives examples of reasoning from observations back to cognitive levels.

The network addresses the three items shown in Figure 3, which appeared as 3, 8, and 17 in the master list. Item 3, (2,1) vs. (3,4), is a level 1 item, since it can be correctly solved by a level 1 strategy: Mixture A has more juice than water, while B has more water than juice. Item 8, (2,6) vs. (3,9), is a level 2 item, since it requires the construction of an equivalence class. Item 17, (3,5) vs. (2,3), is a level 3 item, since a solution that correctly attends to its structure must, in some way, compare ratios.

[Insert Figure 3 about here]

The 21 variables in the network are listed below, with the number of possible values each variable can take in parentheses. Detailed descriptions appear in the following section.

- X₁ Optimal cognitive level (3).
- X₂ Stage within optimal level (3).
- X₃ Optimal stage x level (9).
- X_{4j} Strategy employed on Item j, for j=3, 8, and 17 (10 per item).
- X_{5i} Procedural analysis for Item j (4 per item).
- X_{6j} Understanding of structure of Item j (2 per item).
- X_{7j} Solution of Item j (2 per item).
- X_{8j} Response choice on Item j (3 per item).
- X9i Objective correctness of response choice on Item j (2 per item).

Without constraints, the joint distribution of the variables listed above would be specified as a probability for each of the $3x3x9x(10x4x3x2x2x2)^3$ possible combinations of values—about $7x10^{10}$ of them. Under the assumed network, however,

$$p(X_{1},X_{2},X_{3},X_{4,3},X_{4,8},X_{4,17},...,X_{9,3},X_{9,8},X_{9,17})$$

$$= p(X_{1}) p(X_{2}|X_{1}) p(X_{3}|X_{2},X_{1})$$

$$\times \prod_{j} p(X_{4j}|X_{3}) p(X_{5j}|X_{4j}) p(X_{6j}|X_{5j}) p(X_{7j}|X_{5j}) p(X_{8j}|X_{5j},X_{4j}) p(X_{9j}|X_{8j}) .$$
(6)

As examples, (6) implies conditional independence of item responses, X_{4,3}, X_{4,8}, and X_{4,17}, given a subject's optimal cognitive stage x level, X₃ (although we discuss below relaxing this assumption to account for processes that characterize the adaptive quality of children's strategy choices during the course of testing); and conditional independence of the correctness of the response choice for Item j, X_{9j}, from all other variables given the identity of that response choice, X_{8j}. The most complex of these local relationships in (6) involves only three variables, and the total number of distinct probabilities needed to approximate the full joint distribution is 3+9+81+

3(90+40+120+8+8+6), or 909. As we shall see, many of these relationships are logical rather than empirical, and can be specified without recourse to data.

Figure 4 is the DAG corresponding to (6). Figure 5 is a similar graph from HUGIN, exhibiting for each node the baseline marginal distribution for each variable with bars representing the probabilities for each potential value of a variable. These population base rates were established from the responses of all subjects, as described in the next section. Figure 5 represents the state of knowledge one would have as a new subject from the same population is introduced. As she makes responses, the relevant nodes will be updated to reflect certain knowledge of, say, the correctness of a response or the strategy level used to justify it. This would be represented by a probability bar extending all the way to one for the observed value. This information updates (still imperfect) knowledge about her optimal cognitive level, and expectations about what might be observed on subsequent items.

[Insert Figures 4 and 5 about here]

Instantiating the Network

The initial status of the network is the joint distribution of all the variables. It is specified via (6) in terms of the baseline distribution of any variables without parents, and the conditional distributions of each of the remaining variables given its parents. Béland's classifications of all response explanations of all subjects into stage x level categories were employed, and treated as known with certainty. Explanations of the variables and discussions of the conditional probabilities associated with each follow.

¹ A small proportion of the response strategies could not be classified, because subjects' explanations were either omitted or incomprehensible. These responses were not useful in determining a subject's highest strategy level, but they were included in the following analyses, with "undifferentiated" as a potential value of strategy choice. The proportions for Items 3, 8, and 17 were 2%, 1%, and 11% respectively.

 X_1 : Optimal cognitive level. Each subject was classified as to the stage and level of his or her highest level solution strategy, based on Béland's analyses of all twenty of their response explanations. X_1 denotes their highest *level*, collapsing over stages within levels. Because it has no parents, we need specify only population proportions: .08 for Level 1, .45 for Level 2, and .47 for Level 3.

 X_2 : Stage within optimal level. X_2 breaks down stage membership within levels, so X_1 is its parent. Empirical proportions were employed, leading to the values shown in Table 4. Again these values are based on Béland's classification. Among the subjects whose highest observed level of solution strategy was Level 2, for example, what proportions of these highest strategies were at Stages a, b, and c of Level 2? Stages are meaningful only within levels, so the marginal distribution of X_2 that appears in Figure 5 is not very useful. If X_1 were fixed at a particular value of level, however, the resulting marginal distribution for X_2 would be meaningful, taking the values from the appropriate row of Table 4.

[Insert Table 4 about here]

 X_3 : Optimal stage x level. X_3 is the detailed categorization of subjects into mutually exclusive and exhaustive categories, in terms of levels and stages. It has as parents both level, X_1 , and stage within level, X_2 . The specification of conditional probabilities under this arrangement is logical rather than empirical: The conditional probability of a given stage-within-level value is 1 only if X_1 and X_2 take the appropriate values; otherwise, the conditional probability is zero. This can be seen in Figure 6, where conditioning on X_3 =3b leads to probabilities of one for Level=3 and Stage-within-level=b. Actually no information would be lost by having X_1 and X_2 but not X_3 in the model, or X_3 but not X_1 and X_2 . We have included all of them for interpretive convenience; for example, X_1 is useful for summarizing the "level" information in X_3 , whereas the values for X_3 lie at the same level of detail as those of the Item Strategy variables described below.

Under the "dialectical constructivist" developmental model sketched above, a subject's optimal structure level defines the repertoire of strategies available for solving a given item, as constructed through the changes and transformations that the subjects generated during the course of testing. That is, the optimal state of understanding was constructed by the learners through a series of mental operations that defined the successive levels of conceptualization elaborated to seek the structural properties of the item. Consequently, the optimal structure was not necessarily operationalized before the subjects undertook the task. The dynamics of this process are not modelled in the present example, but will be discussed below. Conversely, the strategy required to solve a given problem was not ultimately at the same level as the subject's optimal stage x level, even when that level has been attained. This observation is taken into account in the present model, through the conditional probability matrices for the following item strategy variables.

[Insert Figure 6 about here]

 X_{4j} : Strategy employed on Item j (j=3, 8, 17). In addition to subjects' optimal strategy stage x level, the particular strategies they employed in the three exemplar items were classified according to stage x level, constituting the variables X_{4j} . The additional value, abbreviated "Ud" in the HUGIN diagrams, stands for "Undifferentiated;" these are the responses which could not be classified. The X_{4j} variables are modelled as conditionally independent, given their common parent X_3 , optimal cognitive level. The conditional probability matrices are presented in Table 5. The following features are noting:

• With a few exceptions, a strategy at any level could be applied to any item. A small number of "logical zeros" appear when the conceptual elements in a given strategy class had no possible correspondents in the structure of an item (e.g., a 2b strategy for Item 17).

- The entire upper right triangle of each matrix is filled with "logical zeros." By definition, it is not possible to observe a response strategy at a higher stage x level than a subject's optimal stage x level.
- The lower left triangle of each matrix was estimated empirically for the most part, by simply entering the proportion of subjects classified in a given optimal stage x level who were classified as employing each of the response strategies for a given item. Probabilities that were logically possible but empirically zero were replaced by small positive probabilities. It can be seen that considerable variation in strategy choice on a given item often existed among subjects with the same optimal level. Among subjects whose optimal stage x level was 3b, for example, about half employed this powerful strategy for the more simply structured Item 8, while about 40% adapted their strategies to the structure of the item and employed a "minimally sufficient" strategy at level 2b. This information appears graphically in Figure 6.

[Insert Table 5 about here]

X5j: Procedural analysis for Item j. These variables summarize the results of the matchups between cognitive strategies and qualitative outcomes. The four possible values are "Success," in which a strategy at the same level as (isomorphic to) the item, or higher, was successfully employed; "Strategic error," in which a strategy was employed which failed to account for the item's structure; "Tactical error," in which a strategy appropriate to the item structure was employed but not successfully executed; and "Computational error," in which the attempt would have been a "Success" except for an error in numerical calculations. The respective X4j variables are the parents. Conditional probabilities corresponding to "Strategic error" are logical, since this outcome is necessary if a strategy that is insufficient vis a vis the item structure is applied, and impossible if a sufficient

strategy is applied.² In the latter case, conditional probabilities are apportioned among "Success," "Tactical error," and "Computational error." Table 6 lists the conditional probability values.

[Insert Table 6 about here]

 X_{6j} : Understanding of structure of Item j. These variables simply collapse from their parents, the X_{5j} s, into the dichotomy of "Understood" or "Misunderstood" the structural properties of the item. In each case, the conditional probability matrix is logical: the probability for "Understood" is one if the procedural analysis is "Success," "Tactical error," or "Computational error," and zero otherwise; the probability for "Misunderstood" is one if the procedural analysis is "Strategic error," and zero otherwise.

 X_{7j} : Solution of Item j. Each of these variables is an alternative collapsing of the corresponding X_{5j} , into the dichotomy of "Succeed" or "Failed." "Failed" occurs if the procedural analysis takes the value of "Strategic error," "Tactical error," or "Computational error." "Success" signifies a correct response through an appropriate strategy.

X8j: Response choice on Item j. These variables are the actual values of subjects' response choices: Mixture A juicier, Mixture B juicier, or equal. The parents of X_{8j} are X_{4j} , strategy, and X_{5j} , procedural analysis. That is, conditional on a particular choice of strategy and the way it is applied on a given item, what are the probabilities of each of the three potential response choices? Table 7 gives the conditional probability table for Item 17 as an example. Recall that whenever a strategy level is insufficient for an item's structure, that strategy level for X_{4j} and "Success" for X_{5j} cannot co-occur. This fact is accounted for in the conditional probability matrix for X_{5j} given X_{4j} , so the corresponding row in X_{8j}

² One exception: two distinct strategies are classified as 1b; one is appropriate for Item 3 but the other is not.

is moot. Entries of equal probabilities appear as spaceholders. Other combinations that were not logically impossible but which few or no subjects exhibited were assigned conditional probabilities that reflected Béland's judgement about likely outcomes, or, if there were no basis for such judgements, equal conditional probabilities.

[Insert Table 7 about here]

Xoj: "Objective" correctness of response on Item j. These variables indicate whether the choices specified in X_{8j} are in fact correct—regardless of how they have been reached. We refer to these as "objective" responses because they are typically the only observations that are available in standard multiple-choice "objective" educational tests. In that context they are thought of as "noisy" versions of the X_{6j}s. The conditional probabilities are logical: for "Correct," the choice that happens to be correct for that item is assigned one and the other two are assigned zero; vice versa for "Incorrect."

Absorbing Evidence

The construction of the network described in the preceding section exemplifies reasoning from causes to effects, as it were. The initial status shown as Figure 5 represents our state of knowledge about a new individual from the same population, beliefs about her likely responses to the sample items and the optimal stage x level we might expect to observe over the course of the twenty-item test. Once she begins to respond, we update our knowledge about observed variables directly, and about still unobserved variables probabilistically. This section offers some examples of how observations update beliefs, particularly with regard to X₁, "optimal cognitive level," and X₂, "optimal stage x level." We focus on some interesting contrasts among the strength and nature of various observations for inferring subjects' cognitive levels.

Recall that these data provide two distinct pieces of evidence on each item, a response choice and an explanation. A first example illustrates a distinction between the value of evidence from the two. Figure 7 shows the network after an incorrect response has been observed to Item

17. The updated status of X_{6,17}, the "Structure understood?" variable for Item 17, indicates an 88% probability that this occurred because of an insufficient strategy and 12% due to inaccurate execution of a sufficient strategy, with probabilities of particular strategy levels shown in X_{4,17}, the "Item strategy" variable for Item 17. Initial beliefs for cognitive levels 1, 2, and 3 in X₁ of 8%, 45%, and 47% have shifted down to 13%, 54%, and 33% (c.f. Figure 5). Expectations for correct responses and understandings of Items 3 and 8 have also been downgraded. Figure 8 shows the additional updating that occurs if we learn this incorrect response was arrived at by a strategy at level 3b, the level isomorphic to the item. Probable explanation for the failure is 20% tactical error, 80% computational error. Belief about overall cognitive level is concentrated on Level 3, and expectations for correct responses to remaining items increase beyond their initial status.

[Insert Figures 7 and 8 about here]

As mentioned above, correct answers to multiple-choice items are typically taken as proxies for correct understandings in educational testing. Test developers avoid items with high "false positive" rates, or probabilities of correct answers by chance or by incorrect reasoning. Figure 9 reveals that Item 17 is just such an item. Of the subjects who responded with the correct choice, fewer than half did so with a strategy that accounted for the true structure of the item! In particular, a quarter of the correct responders employed a level 1b strategy: (3,5) is less juicy than (2,3) because (3,5) has more water. For this reason, a correct response on Item 17 shifts beliefs about optimal level upward only slightly. A correct explanation, on the other hand, would immediately establish certain belief at Level 3.

In contrast, Item 8 is a good multiple-choice item by test theoretic standards.

Figure 10 shows that the overwhelming majority of subjects who answered correctly did so through a correct understanding of the equivalence-class structure of the item.

Interestingly, posterior beliefs shift substantially to level 3 even though only a level 2

strategy is required for correct understanding. This is because nearly all the subjects whose optimal level was 3 understood the structure of Item 8, while less than half of those whose optimal level was 2 did. To further identify whether a correct responder had level 2 or level 3 as an optimal cognitive level would require additional information, such as checking the Item 8 explanation to see if it employed a level 3 strategy (if not, the probability for level 3 would be reduced but not eliminated), or presenting a level 3 item not so prone as Item 17 to false positives (an incorrect response would shift belief to level 2, a correct one to level 3). We note in passing that the second of these options is conditionally independent of the Item 8 choice, given optimal level, whereas the first is not. The DAG (Figure 4) indicates the potential confounding or overlap of information about optimal level from multiple aspects of a response to a given item, due to the presence of the shared "Item strategy" variables linking aspects of information from the same item. One avoids "double counting," or overinterpreting partially redundant information by acting as if it were independent, by properly accounting for the inferential structure of the observations, as demonstrated in this example.

[Insert Figures 9 and 10 about here]

The question of which observation to secure next is addressed by a series of "what if" experiments—a preposterior analysis, in Bayesian terminology. At a given state of knowledge, one can run through the values of a yet unobserved variable, summing the information (in terms of, say, reduced entropy or decreased loss) at each with weights proportional to their predicted probability under current beliefs. The next observation can then be selected to be optimal, in terms of, say, reducing expected loss or reducing expected entropy for a particular unknown variable. This is a straightforward application of statistical decision theory (Raiffa & Schlaifer, 1961).

Comments on the Example

This network provides a simple demonstration compared to the range of potential applications for probabilistic inference about cognitive student models. It does illustrate, however, probability-based reasoning built around structural relationships among cognitive strategies and the qualitatively different states of knowledge under a theory for the acquisition of proportional reasoning.

One of the limitations of this model is that it only provides an explanation of the individual's knowledge organization for a single ability. Consequently, one next step in development might be broadening the scope of the model to accommodate more than one ability—for example, proportional reasoning in a different domain, or something more disparate such as spatial visualization or short-term memory capacity. This can be accomplished by analyzing the structural relationships among individuals' state of learning in different domains. From the cognitive researcher's point of view, an interesting outcome of this study is that it opens up new avenues of exploration in the research of mechanisms and/or processes that lead to the construction of knowledge. Such efforts might create new perspectives for a test theory based on cognitive models. The inferential machinery explored here complements the skill lattice theory Haertel and Wiley (in press) propose as a basis for constructing educational achievement tests.

A more serious limitation is the treatment of subjects' cognitive state. Optimal level was operationalized in the network as the highest strategy level that a subject employed during the course of observation. This is appropriate for inferring the likelihood of a subject's highest level in the entire set knowing just a selected subset of responses. It only tells the whole story, however, under the assumption that a subject's likelihoods of response remained constant over the course of testing—that is, that a subject's toolkit of available cognitive strategies remains unchanged during testing. There is evidence that this is not the case. Cases have been observed in which a subject's previously highest level strategy proves inadequate for a subsequent item, the subject recognizes its inadequacy,

and, in response to the structure of the item, adapts or extends previous strategies or devises new concepts and strategies. Indeed, selecting an item most likely to provoke this kind of restructuring lies at the essence of cognitive-based instruction (Vosniadou & Brewer, 1987)!

The data from which the inference network described above was constructed would support an analysis of this phenomenon, and such work is currently in progress. Figure 11 sketches one direction in which the network described above might be extended to capture key aspects of it. Rather than a single variable expressing a subject's cognitive status throughout the test, there is a distinct variable for each item presented. Cognitive status as it is in effect for Item j depends on the individual's cognitive status as it was before the item was presented and on the structure of Item j itself. The probability that assimilation or accommodation may occur from this interplay is expressed in a new "cognitive processes" variable. We would expect probabilities of adaptive restructuring to be essentially zero when the structure of the item lies below the subject's entering level and low when the item structure is far above her entering level, but maximal when the item lies just beyond what she has been able to handle up to that point.

[Insert Figure 11 about here]

Discussion

A host of practical issues must be addressed in exploring the applicability of probability-based inference, via inference networks, to cognitive assessment. We conclude by mentioning a number of them.

More ambitious student models. The proportional reasoning network discussed above has a very simple representation at its deepest level—a single "optimal level" variable entailing a class of available concepts and strategies. Our challenge was to model the structure of uncertain, partially redundant, sometimes conflicting evidence that observations

convey about the deep variable. A single deep variable is obviously too simple for many practical applications, and we must explore ways to implement student models with many descriptors of knowledge structures, multiple strategy options, and metacognitive and/or affective variables.

The assumed completeness of the network. The inference networks we have discussed are closed systems, which presume to account for all relevant possibilities; i.e., the space of student models is complete. In any application we can hope at best to model the key features distinguishing learners, certainly missing differences that will impact behavior. These differences are modelled as random variation. How does this affect inference? Can we build networks in such a way as to identify unexpected patterns, and to minimize resulting inferential errors?

The nature of student models. Our basic idea is to provide for probabilistic reasoning from observations to student models. This idea can be entertained for any type of student models, but certainly it will prove more useful for some types of student models than others. Characteristics of student models that need to be explored in this connection include model grain-size, and the distinctions between overlay vs. performance models (Ohlsson, 1986), and static vs. dynamic models.

- Grain-size concerns the level of detail at which to model students. As Greeno (1976) points out, "It may not be critical to distinguish between models differing in processing details if the details lack important implications for quality of student performance in instructional situations, or the ability of students to progress to further stages of knowledge and understanding." The grain-size of our example was stage x level. A coarser model would address level only, while a finer model might further differentiate strategies within stages within levels.
- An "overlay" approach to diagnosing knowledge in the context of intelligent
 tutoring systems builds a representation of an expert's knowledge base, and infers

from observed behavior where a student's representation falls short (e.g., C. Frederiksen & Breuleux, 1989). A "performance model" attempts to specify correct and/or incorrect elements of knowledge and application rules in sufficient detail to solve the same problems the student is attempting (e.g., VanLehn, 1990). Our example was a probabilistic version of a simple performance model, as it provides predictions of response probabilities for all items for subjects at all modelled states.

Static models assume a constant knowledge structure during the course of datagathering; dynamic models expect, and attempt to model, changes in the learner along the way. The latter is obviously more ambitious, yet critical to applications such as ITSs in which learning is expected. White and J. Frederiksen's (1987) QUEST system, for example, builds performance models in the domain of simple electrical circuits; the process of instruction is viewed as facilitating the evolution of models, successively shaping student understanding toward that of an expert. Kimball's (1982) calculus tutor utilizes an approach that might be generalized: A student model is built under an assumption of statis during a problem, but the prior distribution for the next problem is modified to reflect the outcome of the experience and a reinforcement model. Our example was static; Figure 11 sketched one possible dynamic extension.

Decision-making and prediction. In the context of medical diagnosis, Szolovits and Pauker (1978, p. 128) point out the necessity of "...introducing some model of disease evolution in time, and dealing with treatment, as diagnosis is hard to divorce from therapy in any practical sense." In the context of education, we are concerned with learning and instruction. The Bayesian inferential machinery, as a component of statistical prediction and decision theory, is natural for this task. What is required is to extend a network to prediction and decision nodes, and to incorporate utilities as well as probabilities into

decision rules. Andreassen, Jensen, and Olesen (1990) illustrate these ideas with a simple example from medical diagnosis. We must lay out the analogous extension in networks for cognitive assessment.

Practical tools. While the inference network approach holds promise for tackling class of problems in cognitive assessment, we are a long way from routinely engineering solutions to particular members of that class. This requires a methodological toolkit of generally applicable techniques and well-understood approaches. Building block models and heuristics are useful, for example, so that each application need not start from scratch. Foundational work on building-block models appears in Schum (1987). Work tailored to the kinds of observational settings and the kinds of psychological models anticipated in educational applications is required. And since simplifications of reality are inevitable, it is important to learn about the consequences of various model violations, and to develop diagnostic techniques for detecting serious ones.

Conclusion

The modelling approach sketched in this paper was motivated by the following consideration:

Standard test theory evolved as the application of statistical theory with a simple model of ability that suited the decision-making environment of most mass educational systems. Broader educational options, based on insights into the nature of learning and supported by more powerful technologies, demand a broader range of models of capabilities—still simple compared to the realities of cognition, but capturing patterns that inform a broader range of instructional alternatives. A new test theory can be brought about by applying to well-chosen cognitive models the same general principles of statistical inference that led to standard test theory when applied to the simple model. (Mislevy, in press).

Probabilistic inference about cognitive student models via inference networks provides a potential framework for a more broadly based test theory. Exploiting conceptual and computational advances in statistical inference, the approach presents an opportunity to extend the achievements of model-based measurement to educational problems cast in terms of contemporary cognitive and educational psychology.

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Table 1 Test Theory Applications with a Cognitive Perspective

- 1. Mislevy and Verhelst's (1990) mixture models for item responses when different examinees follow different solution strategies or use alternative mental models.
- 2. Falmagne's (1989) and Haertel's (1984) latent class models for Binary Skills. Students are modelled in terms of the presence or absence of elements of skill or knowledge, and observational situations demand various combinations of them.
- 3. Masters and Mislevy's (in press) and Wilson's (1989a) use of the Partial Credit rating scale model to characterize levels of understanding, as evidenced by the nature of a performance rather than its correctness. This incorporate into a probabilistic framework the cognitive perspective of Biggs and Collis's (1982) SOLO taxonomy for describing salient qualities of performances.
- 4. Wilson's (1989b) Saltus model for characterizing stages of conceptual development, which model parameterizes differential patterns of strength and weakness as learners progress through successive conceptualizations of a domain.
- 5. Yamamoto's (1987) Hybrid model for dichotomous responses. This model characterizes an examinee as either belonging to one of a number of classes associated with states of understanding, or in a catch-all IRT class. The approach is useful when certain response patterns signal states of understanding for which particular educational experiences are known to be effective.
- 6. Embretson's (1985) multicomponent models integrate item construction and inference within a unified cognitive model. The conditional probabilities of solution steps given a multifaceted student model are given by statistical structures developed in IRT.
- 7. Tatsuoka's (1989) Rule space analyses uses a generalization of IRT methodology to define a metric for classifying examinees based on likely patterns of item response given patterns of knowledge and strategies.

Table 2
Parallels between Inference Networks in Medical and Educational Applications

Medical Application	Educational Application
Observable symptoms, medical tests	Test items, verbal protocols, observers' ratings, solution traces
Disease states, syndromes	States or levels of understanding of key concepts, available strategies
Architecture of interconnections based on medical theory	Architecture of interconnections based on cognitive and educational theory
Conditional probabilities given by physiological models, empirical data, expert opinion	Conditional probabilities given by psychological models, empirical data, expert opinion

Table 3 Stages within Cognitive Levels

Level 1: Conceptual or preoperational

- a Sole comparison of the number of juice glasses, the attribute in both pairs.
- b Appraisal of the dilution effect of the water on the final taste of juice. From this, the order of magnitude became a comparison of the number of water glasses, the *complement* in both pairs.
- c Construction of functional relations between the complementary terms in each pair, establishing between relations in the pair of within relations first constructed.

Level 2: Concrete operational

- a Use of the ratio "one glass of juice for one glass of water" to demonstrate that both terms within each pair were equal.
- b Joint multiplication of both terms within a pair or, otherwise, an operation of comultiplication. (Scalar operator; e.g.,"Both drinks taste alike because there is one glass of juice for three glasses of water, which defines the ratio 1:3 in both pairs.")
- Relationships formed between both terms of each pair, when the first term was divided by the second. (Functional operator, e.g., "The ratio of two glasses of juice for six glasses of water is the same as one glass of juice for three glasses of water. Three times the ratio 1:3 equal three glasses of juice for nine glasses of water. Therefore both drinks taste alike.")

Level 3: Formal operational

- a Either a scalar or functional operator in the between or the within relations.
- B Ratio treatment: The components of the relationships were the attribute and the complement. (E.g., "In Mixture A there are three glasses of juice for five glasses of water, a ratio of 9:15. In Mixture B the ratio is 10:15 juice to water. Therefore, Mixture B tastes juicier.")
- c Fraction treatment: the components of the relationships were the attribute and the quantity of liquid. (E.g., "In Mixture A, of a total of 8 glasses, 3 contain juice, representing a fraction of 15/40. In Mixture B, of a total of 5 glasses, 2 were juice, representing a fraction of 16/40. Therefore, Mixture B tastes juicier.")

Table 4
Conditional Probabilities of Stages within Cognitive Levels

		Stage within Level	
Level	a	b	С
1	.000	.612	.388
2	.582	.345	.073
3	.145	.667	.188

Table 5
Conditional Probabilities of Strategies given Optimal Cognitive Levels

	Strategy Level of Response									
Optimal Level	Ud.	1a	1b	1c	2a	2b_	2c	3a	3b	3c
(Item 3)										
1a	.50	.50	.00	.00	.00	.00	.00	.00	.00	.00
1b	.08	.04	.88	.00	.00	.00	.00	.00	.00	.00
1c	.01	.01	.34	.64	.00	.00	.00	.00	.00	.00
2a	.01	.02	.37	.39	.21	.00	.00	.00	.00	.00
2b	.01	.01	.34	.54	.09	.01	.00	.00	.00	.00
2c	.01	.01	.39	.52	.06	.01	.01	.00	.00	.00
3a	.01	.01	.20	.74	.02	.01	.01	.01	.00	.00
3b	.01	.01	.02	.21	.02	.01	.01	.01	.71	.00
3c	.01	.01	.01	.18	.02	.01	.01	.01	.10	.65
(Item 8)										
1a	.50	.50	.00	.00	.00	.00	.00	.00	.00	.00
1b	.01	.04	.95	.00	.00	.00	.00	.00	.00	.00
1c	.01	.02	.96	.01	.00	.00	.00	.00	.00	.00
2a	.01	.02	.58	.04	.35	.00	.00	.00	.00	.00
2b	.01	.02	.32	.01	.31	.33	.00	.00	.00	.00
2c	.01	.02	.06	.01	.24	.60	.06	.00	.00	.00
3a	.01	.02	.11	.01	.08	.74	.02	.01	.00	.00
3b	.01	.01	.01	.01	.01	.41	.01	.01	.52	.00
3c	.01	.01	.01	.01	.01	.29	.01	.01	.07	.57
(Item 17)										
1a	.50	.50	.00	.00	.00	.00	.00	.00	.00	.00
1b	.07	.01	.92	.00	.00	.00	.00	.00	.00	.00
1c	.04	.01	.94	.01	.00	.00	.00	.00	.00	.00
2a	.03	.01	.43	.06	.47	.00	.00	.00	.00	.00
2 b	.01	.01	.46	.01	.51	.00	.00	.00	.00	.00
2c	.04	.01	.13	.01	.50	.00	.31	.00	.00	.00
3a	.04	.01	.12	.03	.40	.00	.18	.22	.00	.00
3b	.01	.01	.01	.01	.04	.00	.01	.01	.90	.00
3c	.01	.01	.01	.01	.01	.00	.01	.01	.18	.75

Table 6
Conditional Probabilities of Procedural Analysis given Item Strategies

Item Strategy	Success	Strategic Error	Tactical Error	Computational Error
(Item 3)				
Ud	.00	1.00	.00	.00
1a	.00	1.00	.00	.00
1b	.75	.20	.05	.00
1c	.98	.00	.02	.00
2a	.85	.00	.15	.00
2 b	.98	.00	.01	.01
2c	.97	.00	.02	.01
3a	.96	.00	.02	.02
3b	.98	.00	.01	.01
3c	.90	.00	.08	.02
(Item 8) Ud	.00	1.00	.00	.00
1 a	.00	1.00	.00	.00
1b	.00	1.00	.00	.00
1c	.00	1.00	.00	.00
2a	.00	1.00	.00	.00
2b	.98	.00	.01	.01
2c	.00	1.00	.00	.00
3a	.98	.00	.01	.01
3b	.98	.00	.01	.01
3c	.96	.00	.02	.02
(Item 17) Ud	.00	1.00	.00	.00
1a	.00	1.00	.00	.00
1b	.00	1.00	.00	.00
1c	.00	i.00	.00	.00
2a	.00	1.00	.00	.00
2b	.00	1.00	.00	.00
2c	.00	1.00	.00	.00
3a	.70	.00	.10	.20
3b	.95	.00	.01	.04
3c	.97	.00	.02	.01

Table 7

Conditional Probabilities of Item 17 Choice given Item Strategies and Procedural Analysis

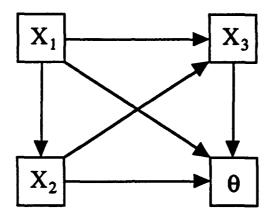
	Procedural		Choice	-
Strategy	Analysis	Mixture A	Mixture B	Equal
Undifferentiated	Success	.33	.33	.33
Undifferentiated	Strategic Error	.13	.12	.75
Undifferentiated	Tactical Error	.33	.33	.33
Undifferentiated	Computational Error	.33	.33	.33
1a	Success	.33	.33	.33
1 a	Strategic Error	.98	.01	.01
1a	Tactical Error	.33	.33	.33
1 a	Computational Error	.33	.33	.33
1b	Success	.33	.33	.33
1 b	Strategic Error	.23	.76	.01
1b	Tactical Error	.33	.33	.33
1b	Computational Error	.33	.33	.33
1c	Success	.33	.33	.33
1 c	Strategic Error	.01	.01	.98
1c	Tactical Error	.33	.33	.33
1c	Computational Error	.33	.33	.33
2a	Success	.33	.33	.33
2a	Strategic Error	.03	.95	.02
2a	Tactical Error .	.33	.33	.33
2a	Computational Error	.33	.33	.33
2 b	Success	.33	.33	.33
2 b	Strategic Error	.33	.33	.33
2 b	Tactical Error	.33	.33	.33
2 b	Computational Error	.33	.33	.33
2c	Success	.33	.33	.33
2c	Strategic Error	.01	.01	.98
2c	Tactical Error	.33	.33	.33
2c	Computational Error	.33	.33	.33

(continued)

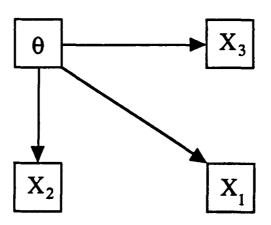
Table 7, continued

Conditional Probabilities of Item 17 Choice given Item Strategies and Procedural Analysis

	Procedural	Choice				
Strategy	Analysis	Mixture A	Mixture B	Equal		
3a	Success	.00	1.00	.00		
3a	Strategic Error	.33	.33	.33		
3a	Tactical Error	.80	.00	.20		
3a	Computational Error	.50	.00	.50		
3b	Success	.00	1.00	.00		
3b	Strategic Error	.33	.33	.33		
3b	Tactical Error	.50	.00	.50		
3b	Computational Error	.38	.00	.62		
3c	Success	.00	1.00	.00		
3c	Strategic Error	.33	.33	.33		
3c	Tactical Error	.90	.00	.10		
3c	Computational Error	.70	.00	.30		



 $p(X_1, X_2, X_3, \theta) = p(\theta | X_3, X_2, X_1) \ p(X_3 | X_2, X_1) \ p(X_2 | X_1) \ p(X_1)$



 $p(X_1, X_2, X_3, \theta) = p(X_1 | \theta) \; p(X_2 | \theta) \; p(X_3 | \theta) \; p(\theta)$

Figure 1
Graphical Representations in the IRT Example

Which mixture will be more juicy—A, B, or both the same?

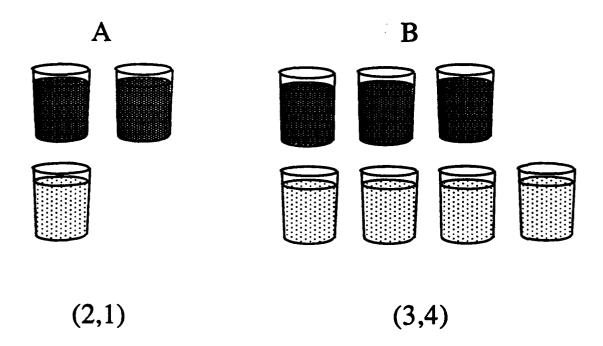


Figure 2

A Sample Juice-Mixing Task

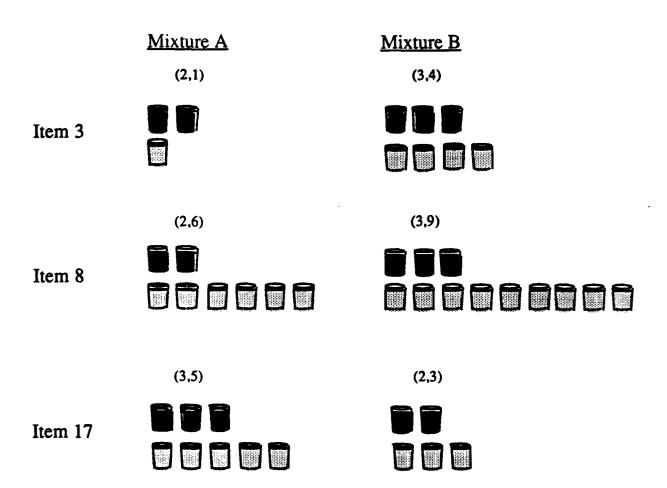
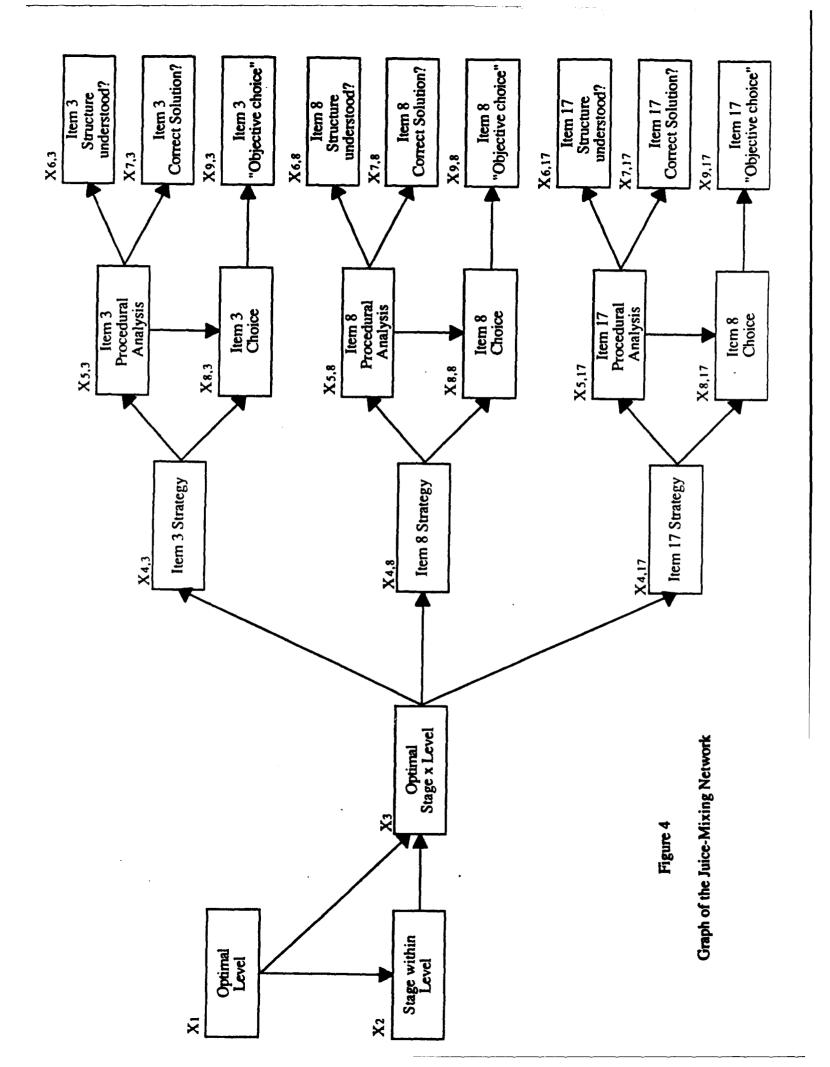


Figure 3

Three Juice-Mixing Tasks



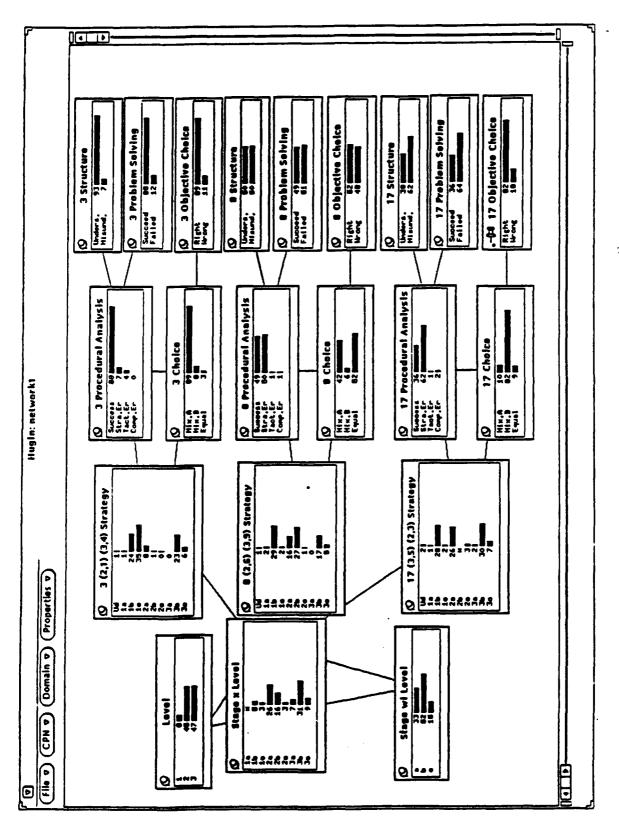


Figure 5

Initial Status, with Marginal Probabilities

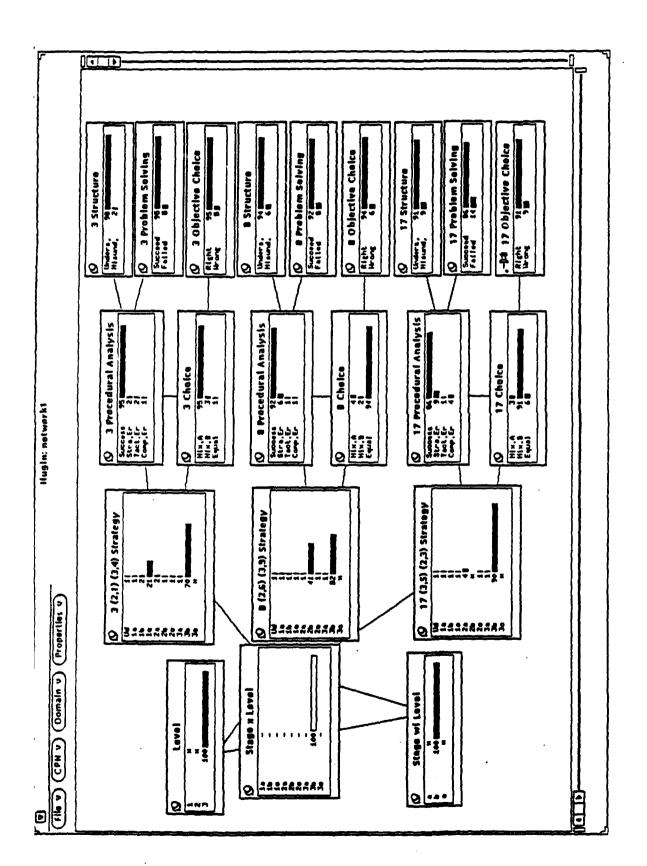


Figure 6

Status Conditional on Optimal Level = 3b

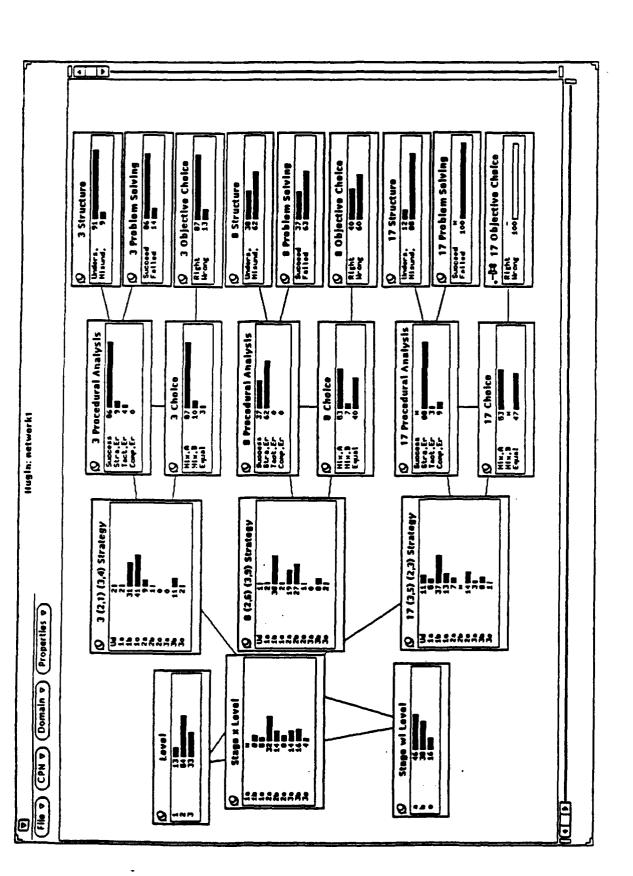
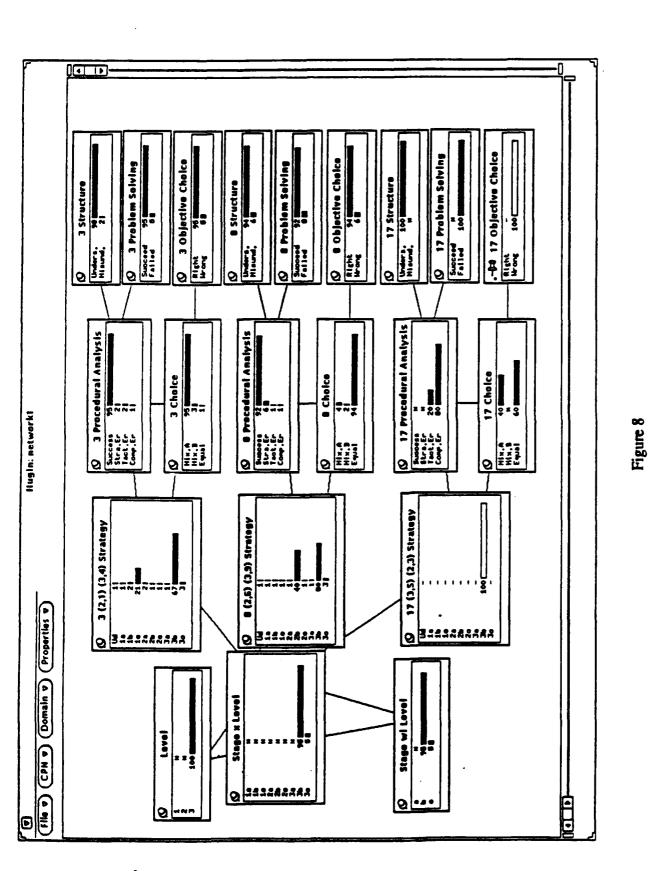


Figure 7

Status Conditional on Item 17 Response Choice = Wrong



Status Conditional on Item 17 Response Choice = Wrong and Item 17 Strategy = Level 3b

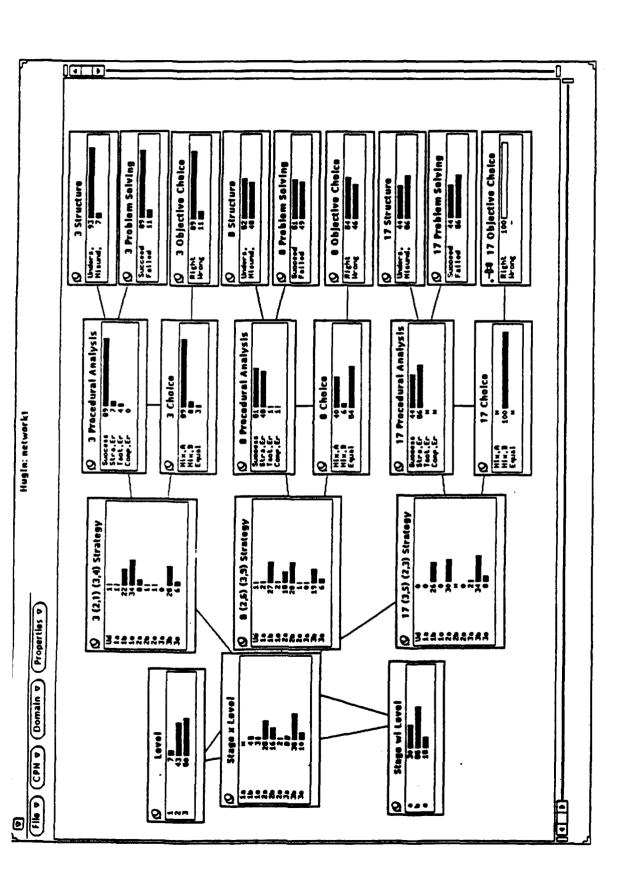


Figure 9

Status Conditional on Item 17 Response Choice = Right

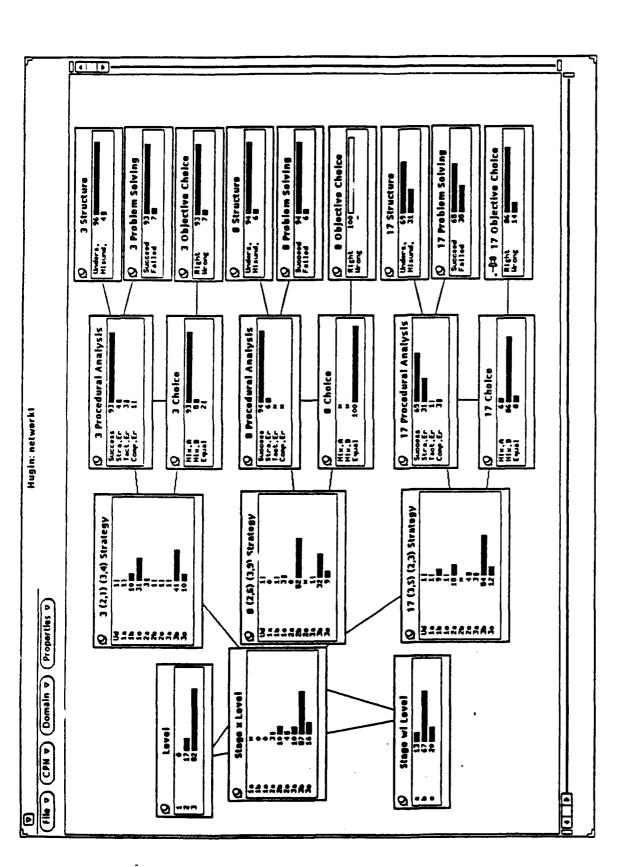
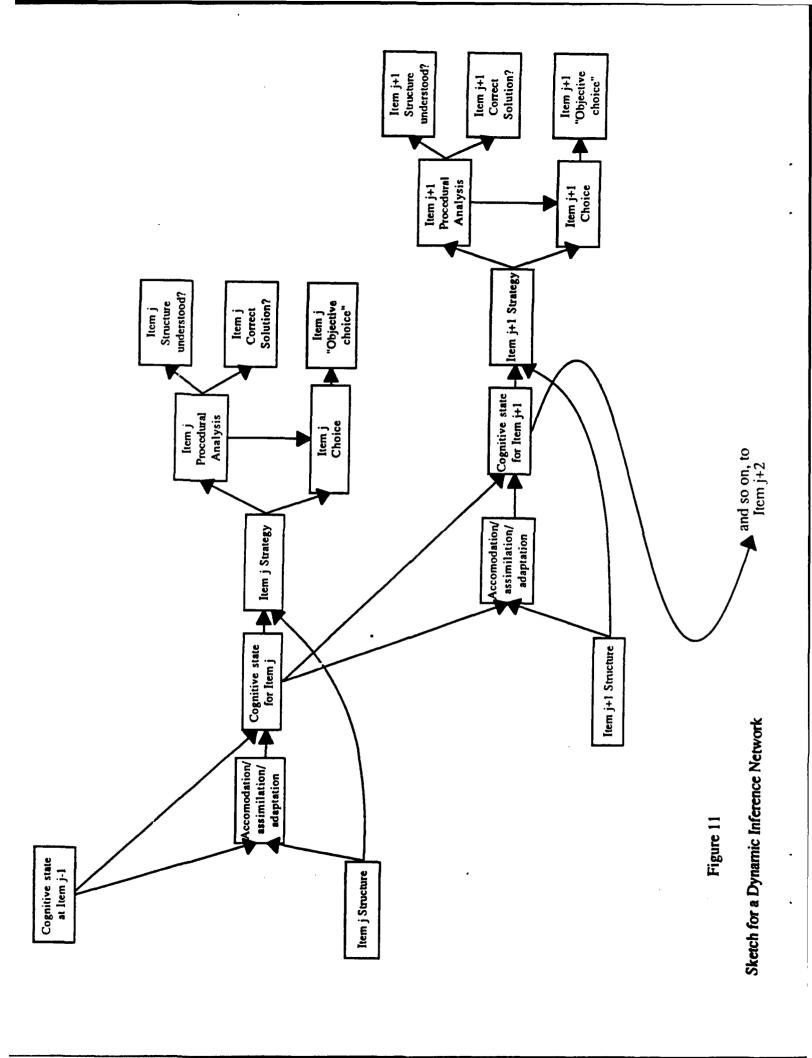


Figure 10

Status Conditional on Item 8 Response Choice = Right



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